

Preliminary Assignment for the research course  
“Peano-type curves obtained from a continuous motion  
of a line segment”

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Solve as much as you can and send me your solutions by email. Don't hesitate to ask me if you need clarification or if you have any questions.

Consider the following *General problem*:

A unit line segment is moved continuously in the plane for a finite amount of time and we consider the (two-dimensional) Lebesgue measure of the trajectory of each point of the unit segment. By parameterizing the unit line segment, we obtain a function  $f: [0, 1] \rightarrow [0, \infty)$ . What functions can we get?

1. Give a precise formalization of the above problem for which  $f$  is always well-defined and finite-valued.
2. Prove that  $f$  can be nonconstant.
3. Prove that  $f$  is always upper semicontinuous.
4. Prove that we can have  $f(0) = 0$  and  $f(1) = 1$ .
5. Prove that the zero set of  $f$  can be a singleton.
6. What else can you say about the zero set of  $f$ ?

Have fun!