Title: Extremal problems in planar graphs

Description: The subject is a new, fast developing area of extremal graph theory. There are two types of basic problems:

(1) determine/estimate the maximum number ex_p(n,F) of edges in a planar graph G of n vertices not containing F as a subgraph.

(2) determine/estimate the maximum number f(n,H) of copies of H in a planar graph G of n vertices

The various constructions of extremal graphs make the subject particularly interesting. The starting point of this subject was the classical result that the maximum number of edges in a planar graph of n vertices is 3n-6 if $n \geq 3$. Many years later, Dowden proved that the maximum number of edges in a planar graph not containing any 4-cycle is at most 12(n-2)/7 and it is sharp for infinitely many values of n. For details, see

C. Dowden, Extremal C_4-free/C_5-free planar graphs, J. Graph Theory 83 (2016), 213– 230.

D. Ghosh, E. Gyori, R. Martin, A. Paulos, C. Xiao, Planar Turan number of the 6-cycle, SIAM J. Discrete Math. 36(3) (2022), 2028–2050.

E. Gyori, X. Wang, Z. Zheng, Extremal planar graphs with no cycles of particular lengths, arXiv:2208.13477 (joint paper with BSM students!)

We plan to consider problems of this type for particular graphs F and H.

Prerequisites: graph theory and combinatorics

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Qualifying problems:

Poblem 1. What is the maximum number of edges in an planar graph of n vertices not containing any four-cycle? Find infinitely many extremal constructions.

Problem 2. What is the maximum number of edges in a planar graph of n vertices not containing any complete subgraph of 4 vertices. Find infinitely many extremal constructions.

Problem 3. What is the maximum number of triangles in a planar graph G of n vertices? Prove (hopefully best) upper bounds, and find constructions (for many values of n) showing that the estimate is sharp.