Bond percolation via the belief propagation algorithm and spectra of Hashimoto matrices

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Qualifying exercises (it suffices to solve two of them):

1. Find the eigenvalues of the adjacency matrix of the following graphs:

   (a) K_n: the complete graph on n vertices, $n \in \mathbb{N}$.

   (b) C_4: the cycle of length 4.

   Please, derive those by hand calculations, numerical results without explanation are not accepted.

2. Show that if $\boldsymbol{A}$ and $\boldsymbol{B}$ are arbitrary $n \times n$ symmetric, positive definite real matrices, then $\boldsymbol{AB}$ (usually not symmetric) has positive real eigenvalues. (Note that $\boldsymbol{A}$ and $\boldsymbol{B}$ usually do not commute, so their eigenvalues are not multiplied together.)

3. With the help of the Euler formula ($e^{it} = \cos t + i \sin t$, $t \in \mathbb{R}$) compute the following complex number: 5^{2+3i}.

   (Here i is the imaginary unit and e is the base of the natural logarithm. $5^{2+3i} = 5^{(2+3i)}$, the exponent is a complex number in algebraic form, and give the result in a closed algebraic form.)