Conjecture & Proof, BSM, 2022 Fall

Assignment #1

To present on Monday, September 12:

1. *Hilbert's hotel.* We have a hotel with countably infinitely many rooms, and all are occupied. What should the management do, if

   (a) a new guest arrives;

   (b) countably infinitely many new guests arrive;

   (c) countably infinitely many buses arrive each bringing countably infinitely many new guests?

2. Is there a bounded set H in the plane which is isometric to a proper subset $S \subsetneq H$?

   (*Isometric* means that there is a distance preserving mapping that maps H to S.)

To present on Wednesday, September 14:

3. Delete one of the corner squares of the chessboard. Is it possible to cover the remaining area by 21 dominoes of size 1×3 ?

4. An invisible flea is jumping on points of the real line. Starting from the origin (at a known moment) it jumps always the same (unknown) real distance to the same (unknown) direction in every minute. We can grasp any segment of length one in every minute. Can we surely catch the flea sooner or later?

5. There are 13 positive integers given with the following property. Deleting any one of them, the rest can be divided into two groups of six numbers with equal sums.

   Does this imply that the given numbers are all equal to each other?

To submit on Wednesday, September 14:

6. Let n and k be positive integers. Prove that $\sqrt[k]{n}$ is either integer or irrational.

Have fun!