EXTREMAL PROBLEMS IN PLANAR AND OUTERPLANAR GRAPHS

Description: The subject is a new, fast developing area of extremal graph theory. There are two types of basic problems:

(1) determine/estimate the maximum number ex_p(n,F) of edges in a(n outer)planar graph G of n vertices not containing F as a subgraph.
(2) determine/estimate the maximum number f(n,H) of copies of H in a(n outer)planar graph G of n vertices

The various constructions of extremal graphs make the subject particularly interesting. Another direction is the combination of these basic type problems: what is the maximum number of copies of H in an n vertex planar graph not containing F as a subgraph. Another possibility is to study the same questions in outerplanar graphs.

The starting point of this subject was the classical result that the maximum number of edges in a planar graph of n vertices is 3n-6 if $n \geq 3$. Many years later, Dowden proved that the maximum number of edges in a planar graph not containing any 4-cycle is at most 12(n-2)/7 and it is sharp for infinitely many values of n. (For details, see C. Dowden, Extremal C_4-free/C_5-free planar graphs, J. Graph Theory 83 (2016), 213– 230.)

Prerequisites: graph theory and combinatorics

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Qualifying problems:

Poblem 1. What is the maximum number of edges in an outerplanar graph of n vertices not containing any four-cycle? And in an outerplanar graph not containing any cycle of length 5.

Problem 2. What is the maximum number of edges in a planar graph of n vertices not containing any triangle or 4-cycle. Find infinitely many extremal constructions!

Problem 3. What is the maximum number of triangles in a planar graph G of n vertices? Prove (hopefully best) upper bounds, and find constructions (for many values of n) showing that the estimate is sharp.

Problem 4. What is the maximum number of paths of length 3 in an outerplanar graph of n vertices.