Causal Vector Autoregression (advisor: *Marianna Bolla*)

Qualifying exercises

1. Consider the symmetric, one-dimensional random walk defined by

$$X_0 = 0, \quad X_t = \sum_{j=1}^{t} \xi_j, \quad t = 1, 2, \dots,$$

   where $\{\xi_j\}$ is a binary process: ξ_js are i.i.d. (independent, identically distributed) and $\xi_j = \pm 1$ with probability $\frac{1}{2} - \frac{1}{2}$.

   Prove that the random walk process $\{X_t\}$ is not weakly stationary.

2. Show that the one-dimensional process

$$X_t = A\cos(\omega t) + B\sin(\omega t), \quad t = 0, \pm 1, \pm 2, \dots$$

   is weakly stationary, where A, B are uncorrelated, standard normal (Gaussian) random variables and $\omega \in [0, 2\pi)$ is a fixed frequency. Find the autocovariance function of the process too.

3. We have a DAG (Directed Acyclic Graph), in which there are no directed cycles. Prove that there is a topological ordering/labeling of the vertices such that for every $i \to j$ edge, the relation $j < i$ holds. (In a directed graphical model, the nodes correspond to random variables, and the $i \to j$ relation means that the random variable corresponding to i is the parent/cause of the one corresponding to j, so one can as well think of labels as ages.)