Distribution of colors in Gallai colorings

Research course, BSM 2019 SUMMER
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A Gallai coloring is an edge coloring of a complete graph K_n that avoids triangles colored with three different colors. A trivial example is a coloring that uses at most two colors. Given integers $e_1 \geq e_2 \geq \cdots \geq e_k$ with $\sum_{i=1}^{k} e_i = \binom{n}{2}$ for some n, does there exist a Gallai k-coloring of K_n with e_i edges in color i? This problem was explored in the paper https://arxiv.org/pdf/1903.04380.pdf The aim of the course is to investigate some problems left open in that paper. A minimum goal should be to find the smallest n for which every sequence $e_1 \geq e_2 \geq \cdots \geq e_5$ with $\sum_{i=1}^{5} e_i = \binom{n}{2}$ can be realized as a color distribution of a Gallai 5-coloring.

Course requirements.

- 1. Understand the statements of the results in the paper arXiv:1903.04380v3.
- 2. Email to me solutions for at least two of the three exercises below, not later than June 10, 2019. If something is not clear, do not hesitate to ask for clarification.

Exercise 1. Prove that in every Gallai coloring of K_n there exists a monochromatic spanning tree (a tree containing all vertices and with all edges of the same color)!
Exercise 2. Prove that in every Gallai coloring of K_n with at least 3 colors, there exists a disconnected color (a color in which the edges do not have a spanning tree)!
Exercise 3. A. Prove that the sequence $8, 3, 3, 1$ can be realized as a color distribution of a Gallai 4-coloring on K_6. B. Prove that the sequence $9, 4, 4, 4$ cannot be realized as a color distribution of a Gallai 4-coloring on K_7.