

## The diameter of large components in $r$ -edge-colorings of $K_n$

The aim of Ramsey theory is to find large monochromatic structures in  $r$ -edge-colorings of a graph  $G$ . The most investigated case is when  $G = K_n$ ; numerous papers and books have been published on this topic.

The diameter of a graph  $G$  is the length of the shortest path between two furthestmost vertices. A *double star* is a tree obtained by connecting the centers of two vertex disjoint stars by an edge. Clearly, a double star has diameter three.

Gyárfás [1] showed that every  $r$ -edge-coloring of  $K_n$  contains a monochromatic connected component of size  $\geq n/(r-1)$  and he also provided examples when this result is tight. Is it possible to say something more about these ‘giant’ monochromatic components? It may even be true that every  $r$ -edge-coloring of  $K_n$  contains a monochromatic double star with at least  $n/(r-1)$  vertices.

**Problem 1.** (*Gyárfás, Problem 4.2 in [2]*) For  $r \geq 3$ , is there a monochromatic double star of size asymptotic to  $n/(r-1)$  in every  $r$ -coloring of  $K_n$ .

A weaker version of the problem reads as follows.

**Problem 2.** (*Gyárfás, Problem 4.3 in [2]*) Given positive numbers  $n, r$ . Is there a constant  $d$  (perhaps  $d = 3$ ) such that in every  $r$ -coloring of  $K_n$  there is a monochromatic subgraph of diameter at most  $d$  with at least  $n/(r-1)$  vertices?

Ruszinkó [4] answered this in affirmative with  $d = 5$ , which has been improved by Letzter [3] to  $d = 4$ . The goal of this research would be to investigate Gyárfás’ Problem for the best possible  $d = 3$  case.

## References

- [1] A. Gyárfás, Partition coverings and blocking sets in hypergraphs (in Hungarian), *Communications of the Computer and Automation Research Institute of the Hungarian Academy of Sciences*, 71 (1977), 62 pp.
- [2] A. Gyárfás, Large monochromatic components in edge colorings of graphs - a survey, *Ramsey Theory Yesterday, Today and Tomorrow*, Progress in Mathematics Series, Vol. 285, ISBN 978-0-8176-8091-6, Birkhäuser, 77-96.
- [3] S. Letzter, Large Monochromatic Triple Stars in Edge Colourings, *Journal of Graph Theory*, 80(4), (2015), 323-328.

- [4] M. Ruszinkó, Large components in  $r$ -edge-colorings of  $K_n$  have diameter at most five, *Journal of Graph Theory*, 69(3) (2012), 337-340.

Prerequisites: basic combinatorics, graph theory, Ramsey theory.

Best for: students interested in combinatorics, discrete mathematics, computer science or information theory.

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Assignment for the first week: read and try to digest the linked papers, starting with of Ruszinkó.