Preliminary assignment for the research course "Hausdorff dimension of the graphs of injective functions"

BSM, 2019 Spring

At least two of these exercises should be done by the Welcome Party, and all of them before our first meeting. If you send your solutions by e-mail to `tamas.keleti@gmail.com`, you can get feedback before we meet.

Let C be the Cantor set.

1. Find the definition(s) of Hausdorff dimension in the internet, and (directly from one of the equivalent definitions) prove that $C \times C$ has Hausdorff dimension at most $\log 4 / \log 3$.

2. Study and understand the Mass Distribution Principle and its direct applications in K. Falconer: *Fractal Geometry: Mathematical Foundations and Applications* (see e.g. `http://wwwf.imperial.ac.uk/~jswlamb/M345PA46/F03%20chap%201-4.pdf`) pages 60-61, and using this method prove that the Hausdorff dimension of $C \times C$ is at least $\log 4 / \log 3$.

3. Prove that if f is a continuous and injective real function on $[0, 1]$ then the Hausdorff dimension of the graph of f must be 1.

Have fun!