The diameter of large components in r-edge-colorings of K_n II

The aim of Ramsey theory is to find large monochromatic structures in r-edge-colorings of a graph G. The most investigated case is when $G = K_n$; numerous papers and books have been published on this topic.

The diameter of a graph G is the length of the shortest path between two furthermost vertices. A *double star* is a tree obtained by connecting the centers of two vertex disjoint stars by an edge. Clearly, a double star has diameter three.

Gyárfás [1] showed that every r-edge-coloring of K_n contains a monochromatic connected component of size $\geq n/(r-1)$ and he also provided examples when this result is tight. Is it possible to say something more about these 'giant' monochromatic components? It may even be true that every r-edge-coloring of K_n contains a monochromatic double star with at least $n/(r-1)$ vertices.

Problem 1. *(Gyárfás, Problem 4.2 in [2]) For $r \geq 3$, is there a monochromatic double star of size asymptotic to $n/(r-1)$ in every r-coloring of K_n.*

A weaker version of the problem reads as follows.

Problem 2. *(Gyárfás, Problem 4.3 in [2]) Given positive numbers n, r. Is there a constant d (perhaps $d = 3$) such that in every r-coloring of K_n there is a monochromatic subgraph of diameter at most d with at least $n/(r-1)$ vertices?*

Ruszinkó [4] answered this in affirmative with $d = 5$, which has been improved by Letzter [3] to $d = 4$. During a BSM research course of spring 2019 Ruszinkó and two students showed that Gyárfás' conjecture in terms of diamater for several values of r is best possible.

The goal of this research would be to explore Gyárfás' problems further.

References

[1] A. Gyárfás, Partition coverings and blocking sets in hypergraphs (in Hungarian), *Communications of the Computer and Automation Research Institute of the Hngarian Academy of Sciences,* 71 (1977), 62 pp.

[2] A. Gyárfás, Large monochromatic components in edge colorings of graphs - a survey, *Ramsey Theory Yesterday, Today and Tomorrow,* Progress in Mathematics Series, Vol. 285, ISBN 978-0-8176-8091-6, Birkhäuser, 77-96.

[3] S. Letzter, Large Monochromatic Triple Stars in Edge Colourings, *Journal of Graph Theory,* 80(4), (2015), 323-328.

[4] M. Ruszinkó, Large components in r-edge-colorings of K_n have diameter at most five, *Journal of Graph Theory*, 69(3) (2012), 337-340.

Prerequisites: basic combinatorics, graph theory, Ramsey theory.

Best for: students interested in combinatorics, discrete mathematics, computer science or information theory.

Professor: Dr. Miklós Ruszinkó.

Assignment for the first week: read and try to digest the linked papers, starting with the one of Ruszinkó.