# COURSE DESCRIPTION FOR "CLASSICAL ANALYTIC NUMBER THEORY"

## 1. The subject

In the XIXth century, it turned out that analytic methods can be applied to prove statistical results on arithmetically defined quantities (occasionally, these statistical results give rise to qualitative ones, e.g. for primes in arithmetic progressions). The goal of this class is to learn the most important properties of the Riemann zeta function and the Dirichlet $L$-functions, and to understand how one can apply them to obtain statistical information on primes.

Prerequisites: introductory number theory, elements of complex analysis and/or Fourier analysis (these last two will be briefly reviewed).

## 2. Topics

**Elementary theory.** The zeta function for $s > 1$. Theorems of Chebyshev and Mertens.

**Dirichlet characters.** Elementary properties. Nonvanishing of $L(1, \chi)$. Dirichlet's theorem on primes in arithmetic progressions.

**The prime number theorem.** The zeta function as a complex function and its analytic properties. Zero-free region. The prime number theorem.

**The prime number theorem for arithmetic progressions.** The Dirichlet $L$-functions as complex functions and their analytic properties. Zero-free regions and the Siegel zero. The prime number theorem for arithmetic progressions.

**Character sums.** The Pólya-Vinogradov theorem. Burgess's bound.