The rate of disjointly superimposed codes

This is the proposed research problem to explore:

A set of binary $0-1$ sequences $\mathcal{C}$ of length n is an r-superimposed code, if for arbitrary sub-collections $\mathcal{A} \neq \mathcal{B}$ of $\mathcal{C}$ such that $\mathcal{A} \leq r$ and $\mathcal{B} \leq r$, $\bigvee\limits_{a \in \mathcal{A}} a \neq \bigvee\limits_{b \in \mathcal{B}} b$.

($\bigvee$ stands for the bit-wise OR function.)

To determine the maximum size $f(n, r)$ of an r-superimposed code seems to be hard, the problem is still widely open (in spite of attempts of, e.g., Erdős, Frankl and Füredi). A related, and I think easier problem is to determine the maximum size $f'(n, r)$ of a disjointly r-superimposed code, where the above requirenment is relaxed only for disjoint sub-collections $\mathcal{A} \cap \mathcal{B} = \emptyset$.