Game Theory

Course Information

Instructor: Patroklos Benatos
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Office hours: By appointment after class or over the internet.
Course page: https://pbenatos.web.elte.hu
Prerequisites: You need to be familiar with the style of *rigorous* and *precise* arguments. The main mathematical prerequisite is familiarity with the basic elements of *naive set theory*. Additionally, in the second half of the semester, you will need the basic elements of *elementary probability theory* and *vectors* and *matrices* (i.e.: vector spaces and linear functions); nevertheless, I can provide a self-contained intro for those in need before we start using them. Some more details are on the "Is this Course for You?" page.
Approach: The overall key characteristic of the course is the emphasis on the *depth of understanding* rather than on the quantity of material. In addition to your knowledge of the field, the class also aims at contributing to your *ability to work in a team* (see Format).
Format: The class *combines* the *lecture* and the *problem solving session* format. Problem solving sessions are where we discuss many of the homework problems and any questions you may have regarding the material. There is on average one problem solving session hour for every two or three lecture classes. You will be working in a team of 2-3 students on some of the homework problems and teams will be able to ask questions at the problem solving sessions from the instructor. We rotate team members as the semester progresses so that you get to know your fellow classmates.
Text: The course does not follow any textbook but rather it is a mesh of several textbooks combined with the instructor's own approach to how the material can be structured and explained. Textbooks useful for background reading are listed on the course homepage at http://pbenatos.web.elte.hu/?p=216.
Class notes covering about 90% of what is discussed will be available for download in pdf format.
Exams: There is a midterm and a final written (take home) exam.
Grading: Your grade is determined by homework assignments (50%) and a midterm and a final exam (50%) (see Requirements and Grading Policy below for details). Class activity also contributes to your grade.
Syllabus: See next page. I consider the syllabus as a tentative plan that can be (somewhat) adjusted based on how we progress in accordance with the aim that proper understanding is more important than quantity of material.

Requirements and Grading Policy

Requirements

1. Homework assignments

There will be 8 ± 1 problem sets assigned during the semester, available on-line in advance of a problem solving session. I will assign 1-2 problems to each team and you will need to prepare with your teammates to do these problems. Teams will be able to ask detailed questions at the problem solving session and if the questions are "good enough" I will not avoid solving the particular homework problem completely. After the session you have to write up your solutions for submission ***individually***; nevertheless, while you *may* ***not pass on anything in written form*** to a fellow classmate after the session, you may help him/her ***verbally***. Credit is given for ***how you show your understanding*** of a solution and ***not*** for some (numeric) answer being correct or not.

You can submit your homework in traditional paper or in electronic format (and can also write up your solutions by hand, scan it in as a pdf document, and submit that electronically). For electronic submissions, file name template is **LastnameFirstname_GMTP#_BSM2018S.pdf**, where # is the problem set number (so the file format is **pdf**).

2. Midterm and Final Exam

These are standard (take-home) written tests to be solved individually.

Grading Policy

Your total points from the problem sets and both the midterm and final exam will be projected onto a 100 scale. These two results will be added up with weights 0.5 and 0.5 respectively to produce your result for the course on a 100 scale. Letter grades are then assigned as follows:

$85 \leq A$

$75 \leq B < 85$

$65 \leq C < 75$

$55 \leq D < 65$

$F < 55$

If your points fall within ±2 points of these limit points, there is fine structure with X^+ and X^- (exceptions: above 96 is A+ and under 55 there is no F+).

Topics Covered

The topics below cover the basics of **competitive** game theory; **cooperative** game theory is only touched upon and has to be covered in another course.

1. Introduction: What is (competitive) Game Theory?

We discuss several real-life examples to get an idea what kind of situations competitive game theory tries to model and what the basic building blocks of such model should be.

2. Basic Models: the Strategic and the Extensive Form and Players

We first build the classification (taxonomy) of the situations the theory treats and the player models it uses, then proceed to the two basic models the theory has for these situations: the *strategic* and (the simplest form of) the *extensive* form. Additionally, we discuss two special properties a game might have: symmetry and strict competitiveness.

3. Game Played: Dominance, Security and Best Response; Pure Equilibrium, Backward Induction, Subgame Perfect Equilibrium, Equilibrium Selection and Common Knowledge
We examine what we think should happen as a game is played by our player models from *two perspectives*: one from the "outside", i.e. what we, as clever scientists think should be some "end state" of the game, and another from the "inside", i.e. what should happen if we follow the logic of the player models from inside the game. Will the results match? Furthermore, how do the special properties a game might have affect the play of the game?

4. Game Theory and Real Life: Applications, Experiments, Predictions
We examine how our theoretical findings so far can be relevant to real life – among other things, what is a "social dilemma" and what is the "Small World Principle".

5. Basic Models: Decision under Uncertainty, Rationality, Utility
We make an important step further in the theory with the mathematical formulation of several notions, which, until this point, have only been "floating around": what is a *preference* relation, a von Neumann – Morgenstern utility function and what do we mean by a *rational* decision-maker in certain and uncertain situations?

6. Basic Models: The Strategic and the Extensive Form with External Chance Events, Information Sets and Randomized Strategies
We return to the basic models: with the model of rational decisions under uncertainty available, we can add randomized strategies and external chance events. Additionally, we include the modeling of the players' knowledge of the flow of the game as "information sets". We discuss perfect recall and Kuhn's theorem: in a finite extensive game of prefect recall, what type of random strategy should we use, behavioral or mixed?

7. Game Played: Chance events, Imperfect Information, Randomized Strategies
How do chance events and randomized strategies influence game play? We find that while the conceptual framework we learnt in Chapter-3 does not change (we do need to re-phrase the notions of domination, iterated elimination of dominated strategies, security strategies, best response functions, mutual best responses and equilibrium in mixed strategies), there are quite a few important additions; especially in computing equilibria. Additionally, we briefly discuss game play in imperfect information extensive games: the notion of sequential equilibrium.

The topics listed above, together with the problem-solving sessions, are more than enough to cover in the one semester at the BSM. Additionally, if time permits, one or two of the following topics will be touched upon:
Evolutionary Game Theory (Evolutionarily Stable Strategy, Replicator Equation)
Games of Incomplete Information and Mechanism Design (Sequential and Bayesian Equilibrium, Revelation Principle**)**
Repeated Games (Strategies, Payoffs, Folk Theorems)
Recursive and Stochastic Games (Stationary Strategies, Shapley's Theorem)