Finding sails for summer 2017

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March 21, 2017

The **sail** is a configuration of four triples $123, 145, 167, 357$ on $\{1, 2, \ldots, 7\}$ (you can probably find a drawing that justifies its name). The most interesting open question from our summer research program in 2016 is the following.

Problem 1. *Is it true that in any 2-coloring of the blocks of a sufficiently large Steiner triple system (STS), there is a monochromatic sail, i.e. three blocks of the same color, forming a sail.*

For necessary definitions se [1]. We try to make some advances, at least to prove it for some infinite family of Steiner triple systems. The most hopeful candidates are the ones with a nice structure:

- projective STS $P(n)$: Points are all $0, 1$ vectors of length $n + 1$ without the all zero vector, three vectors forming a block if their binary sum is zero at each coordinate.
- affine STS $A(n)$: Points are all $0, 1, 2$ vectors of length n, three vectors forming a block if they are all different or all agree in each coordinate.

Requirement for the course: Familiarity with [1] (you can get it from BSM home page), solutions of exercises below, emailed to me at least one day before the opening party of the summer semester.

Exercise 1. Give 2-colorings of $P(2)$ and $A(2)$ without monochromatic sail.
Exercise 2. Give 2-coloring of a Steiner triple system on 13 points without monochromatic sail.

References

[1] Elliot Granath, András Gyárfás, Jerry Hardee, Trent Watson, Xiaoze Wu, Ramsey theory on Steiner triples, to appear.

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