Ramsey problems on Steiner triple systems

Research Experience Course

A *Steiner triple system* $T = (V, B)$ is a set V together with a set B of 3-element subsets of V, called *blocks* such that every distinct pair $v, w \in V$ is in *precisely one block* of B. We denote by $STS(n)$ any Steiner triple system with $|V| = n$. A *2-coloring* of an $STS(n)$ is an assignment of red or blue colors to the blocks.

Exercise 1. *How many blocks are in $STS(n)$?*

Exercise 2. *Prove that if $STS(n)$ exists then $n \in \{1, 3\} \pmod 6$.*

Exercise 3. *Find the (unique) $STS(7)$ and $STS(9)$.*

A *partial triple system* $T = (V, B)$ is a set V together with a set B of 3-element subsets of V, called *blocks* such that every distinct pair $v, w \in V$ is in *at most* one block of B. We denote by $PTS(n)$ any partial triple system with $|V| = n$.

A partial triple system H has the *Ramsey property* if the following is true: there is an $n_0 = n_0(H)$ such that for $n \geq n_0$ in every 2-coloring of any $STS(n)$ there is a red or a blue copy of H. The aim of the project is to find as many nice H as possible.

Exercise 4. *Prove that partial triple systems with two blocks have the Ramsey property.*

Solutions will be collected at the opening party or can be sent before that time by email to gyarfas@renyi.hu.

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