Assignment for the first week: Solve the following.

Exercise 1. *Fix*

- $t_0, t_1, t_2, t_3 \in \mathbb{C}$ *all distinct*

- $q_1, q_2 \in \mathbb{C}$ *distinct from the all the elements* t_j

- $u_0, u_1, u_2, u_3 \in \mathbb{C}$ *sufficiently generic (see the assumption below)*

- $p_1 \in \mathbb{C}$ *arbitrary.*

Assume there exists a single solution $(\alpha_1, \alpha_2, \alpha_3)$ *of the linear system*

$$\alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 = u_0$$
$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$
$$\alpha_1 t_1 + \alpha_2 t_2 + \alpha_3 t_3 = t_0.$$

Solve the equation

$$\alpha_1 u^1 \frac{t_1 - q_2}{t_1 - q_1 + u^1(p_2 - p_1)} + \alpha_2 u^2 \frac{t_2 - q_2}{t_2 - q_1 + u^2(p_2 - p_1)}$$
$$+ \alpha_3 u^3 \frac{t_3 - q_2}{t_3 - q_1 + u^3(p_2 - p_1)} = u^0 \frac{t_0 - q_2}{t_0 - q_1 + u^0(p_2 - p_1)}.$$

for $p_2 \in \mathbb{C}$ *and discuss the obtained solution from the following perspectives: number of solutions* p_2, *special case* $q_2 = q_1$.