Title: **What is unavoidable – Forbidden Configurations**

Description: Consider a family $\mathcal{F}$ of subsets of $\{1, 2, \ldots, n\}$. Balogh and Bollobás proved that if $|\mathcal{F}| > f(k)$, then there exists a subset $A \subset \{1, 2, \ldots, n\}$ such that $\mathcal{F}|_A$ is either a k-star, complement of a k-star or a chain of length k. The interesting point here is that $f(k)$ does not depend on n. Another basic result is Sauer's theorem stating that if $|\mathcal{F}| > \binom{n}{k-1} + \binom{n}{k-2} + \ldots + \binom{n}{0}$ then there exists a k-element subset $K \subset \{1, 2, \ldots, n\}$ such that $\{K \cap F \colon F \in \mathcal{F}\} = 2^K$. Both of these results state that if a set system is large enough, then it cannot avoid certain traces. Our interest is in when does a given trace or family of traces appear?

The topic is extremal hypergraph theory formulated mostly in the language of 0-1 matrices, since that is the most convenient way. We say that a 0-1 matrix A has F as a configuration if there is a submatrix of A, which is a row and column permutation of F. This concept is also called a *trace* or *subhypergraph* depending upon whether the context is set systems or hypergraphs.

The fundamental question is that for given F (or sometimes for given collection $\mathcal{F}$ of configurations) what is the largest possible number of columns of a simple $m \times n$ 0-1 matrix A on given number of rows without having F (or any member of $\mathcal{F}$) as configuration, in notation $\text{forb}(F, m)$ or $\text{forb}(\mathcal{F}, m)$. A 0-1 matrix is simple if it has no repeated columns. Considering columns as characteristic vectors of subsets of an n element set matrix A describes a hypergraph, or set system.

In this project you will get acquainted with basic proof ideas of the area, such as "standard induction", shifting, applications of graph theory and linear algebra. The goal is to get results on particular forbidden configurations. For more info check
`http://www.combinatorics.org/ojs/index.php/eljc/article/view/DS20`

Prerequisites: basic combinatorics, possibly linear algebra;

Best for: students who intend to do research in combinatorics

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