Conjecture and Proof   C&P

Instructor: Dr. Róbert FREUD

Text: M. Laczkovich, Conjecture and Proof + handouts

Prerequisite: Introductory math courses

We try to follow the instructions of Paul Erdős: Conjecture and prove! We shall see also some of his favorite problems and enjoy some proofs from — what he called — "The Book". According to his spirit, the course intends to show the many and often surprising interrelations between the various branches of mathematics (algebra, analysis, combinatorics, geometry, number theory, and set theory), to give an introduction to some basic methods of proofs via the active problem solving of the students, and also to exhibit several unexpected mathematical phenomena. Among others we plan to find answers to the following questions:

- Can the real function $f(x) = x$ be written as the sum of (finitely many) periodic functions?
- Can you chop a cube by plane cuts into finitely many polyhedra and reassemble a regular tetrahedron (using each part once)?
- Can you "cut" a ball into finitely many (not necessarily "nice") subsets and "reassemble" two(!) copies of the original ball?
- Can you color the numbers $1, 2, \ldots, 2015$ using two colors so that for no a and $d > 0$ can $a, a + d, a + 2d, \ldots, a + 17d$ all have the same color?
- Can every integer large enough be represented as the sum of 6 sixth powers, i.e. is $n = x_1^6 + \ldots + x_6^6$ solvable in nonnegative integers x_i if n is large enough?

Topics:

Pigeonhole principle, counting arguments, invariants for proving impossibility, applications in combinatorics and number theory.

Irrational, algebraic and transcendental numbers, their relations to approximation by rationals and to cardinalities.

Vector spaces and fields, field extensions, geometric constructions (doubling the cube, squaring the circle, trisecting the angle, constructing regular polygons), Hamel bases, Cauchy's functional equation, finite fields.

Isometries, geometric and paradoxical decompositions: Bolyai–Gerwien theorem, Hilbert's third problem, Hausdorff paradox, Banach–Tarski paradox.