### Game Theory Course Information

Instructor:	Patroklos Benatos
E-mail:	pbenatos@outlook.com
Office hours:	By appointment, usually after class or over the internet.
Course page:	https://pbenatos.web.elte.hu
Prerequisites:	You need to be familiar with the style of <i>rigorous</i> and <i>precise</i> arguments. The only mathematical prerequisite is familiarity with the basic elements of naive set theory. In the second half of the semester, we use some elementary probability theory and vectors and matrices. Further details are on the "Is this Course for You?" page.
Approach:	The overall key characteristic of the course is the emphasis on the <i>depth of understanding</i> rather than on the quantity of material. In addition to your knowledge of the field, the class also aims at contributing to your <i>ability to work in a team</i> and your <i>presentation skills</i> (see Format).
Format:	The class <i>combines</i> the <i>lecture</i> and the <i>problem solving seminar</i> format. Problem solving seminars are where we discuss some of the homework problems and questions you may have regarding the material. There is approximately one problem solving seminar hour for every two or three lecture classes. You will be working in teams of 2-3 students on one or two homework problems and teams will need to come to these seminars prepared with solutions (or questions) to present them to the other students in the class. We rotate team members as the semester progresses so that you get to know your fellow classmates.
Text:	The course does not follow any textbook but rather it is a mesh of several textbooks combined with the instructor's own approach as to how the material can be structured and explained. Textbooks useful for background reading are listed on the course homepage at <a href="http://pbenatos.web.elte.hu/?p=216">http://pbenatos.web.elte.hu/?p=216</a> . Class notes covering about 90% of what is discussed will be available for download in pdf format.
Exams:	There is a midterm and a final written (take home) exam. The problems on the exams are closely related to the homework problems.
Grading:	Your grade is determined by homework assignments (50%) and a midterm and a final exam (50%) (see Requirements and Grading Policy below for details). Class activity also contributes to your grade.
Syllabus:	See next page. I consider the syllabus as a tentative plan that can be (somewhat) adjusted based on how we progress in accordance with the aim that proper understanding is more important than quantity of material.

### **Requirements and Grading Policy**

#### Requirements

#### 1. Homework assignments

There will be  $8 \pm 1$  problem sets assigned during the semester; downloadable prior to a problem solving seminar. I will assign 1-2 problems to each team and you will need to prepare with your team to present these to your fellow students at the seminar (I will always be available for questions via e-mail prior to the seminar and will also help you during the presentation if needed, of course). After the seminar, however, you have to write the solutions up for submission *individually*. Credit is given for *how you show your understanding* of a solution and not for some (numeric) answer being correct or not. You can submit your homework in traditional paper or in electronic format and can also write up your solutions by hand, scan it in as a pdf document, and submit that electronically. For electronic submissions, file name template is LastnameFirstname\_GMTP#\_BSM2016F.pdf, where # is the problem set number (so the file format is pdf).

#### 2. Midterm and Final Exam

These are standard (take-home) written tests to be solved individually.

#### **Grading Policy**

Your total points from the problem sets and both the midterm and final exam will be projected onto a 100 scale. These two results will be added up with weights 0.5 and 0.5 respectively to produce your result for the course on a 100 scale. Grades are then assigned as follows:

 $\begin{array}{l} 85 \leq A \\ 75 \leq B < 85 \\ 65 \leq C < 75 \\ 55 \leq D < 65 \\ F < 55 \end{array}$ 

If your points fall within  $\pm 2$  points of these limit points, there is fine structure with X<sup>+</sup> and X<sup>-</sup> (exceptions: above 96 is A+ and under 55 there is no F+).

### Syllabus

The topics below cover the basics of **competitive** game theory; **cooperative** game theory is only touched upon and has to be covered in another course.

#### **1. Introduction: What is (competitive) Game Theory?**

We discuss several real-life examples to get an idea what kind of situations competitive game theory tries to model and what basic building blocks such model should have.

#### 2. Basic Models: the Strategic and the Extensive Form and Players

We first build the classification (taxonomy) of the situations the theory treats and the player models it uses, then proceed to the two basic models the theory has for these situations: the *strategic* and (the simplest form of) the *extensive* form. Additionally, we discuss two special properties games might have: symmetry and strict competitiveness.

## **3.** Game Played: Dominance, Security, Common Knowledge, Pure Equilibrium, Backward Induction, Subgame Perfect Equilibrium and Equilibrium Selection

We examine what we think should happen as a game is played by our player models from *two perspectives*: one from the "outside", i.e. what we, as clever scientists think should be some "end state" of the game, and another from the "inside", i.e. what should happen if we follow the logic of the player models from inside the game. Will the results match?

#### 4. Game Theory and Real Life: Applications, Experiments, Predictions

We examine how our theoretical findings can be relevant to real life – among other things, what is the "Small World Principle" and what is a "social dilemma".

#### 5. Basic Models: Decision under Uncertainty, Rationality, Utility

We make an important step further in the theory with the *mathematical formulation* of several notions, which, until this point, have only been "floating around" in our discussion: what is a *preference* relation and what do we mean by a *rational* decision-maker in an uncertain situation?

# 6. Basic Models: The Strategic and the Extensive Form with Randomized Strategies and External Chance Events; Information Sets

We return to the basic models and examine what amendments we need to make with randomized strategies and external chance events. In addition, we also include the modeling of the players' knowledge of the flow of the game as "information sets". We also discuss perfect recall and Kuhn's theorem: in a finite extensive game of prefect recall, which should we use, behavioral or mixed strategies?

# 7. Game Played: Strategic Form with Mixed Strategies and Extensive Form with Perfect Information

How do mixed strategies influence game play in the strategic form? We find that while the basic structure we learnt in Chapter-3 does not change (we do need to re-phrase the notions of domination, iterated elimination of dominated strategies, security strategies, best response functions, mutual best responses and equilibrium in mixed strategies), there are quite a few important additions; especially in computing equilibria.

Additionally, we briefly discuss game play in imperfect information extensive games: the notion of sequential equilibrium.

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Repeated Games (Strategies, Payoffs, Folk Theorems)
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Recursive and Stochastic Games (Stationary Strategies, Shapley's Theorem)

Note: a more detailed syllabus is at <u>https://pbenatos.web.elte.hu/?p=491</u>.

The topics listed above, together with the problem-solving seminars, are more than enough to cover in the one semester at the BSM. Additionally, if time permits, one or two of the following topics will be touched upon:

**Evolutionary Game Theory** (Evolutionary Stable Strategy, Replicator Equation) **Games of Incomplete Information and Mechanism Design** (Bayesian Equilibrium, Revelation Principle)