2-colorable triple systems

Research Project

A triple system T = (V, E) (3-uniform hypergraph) is a set V together with a set E of 3-element subsets of V. A proper k-coloring of a triple system T is an assignment of "colors" from $\{1, 2, ..., k\}$ to the elements of V so that on every triple of E at least two different colors are assigned. The chromatic number of a triple system T = (V, E) is the smallest k such that T has a proper k-coloring.

Exercise 1. Find a 3-chromatic T = (V, E) with |V| as small as possible.

Exercise 2. Find a k-chromatic T = (V, E) with |V| as small as possible.

Exercise 3. Assume that any two triples in T = (V, E) has non-empty intersection. Prove that T is 3-colorable! Is it true that T is 2-colorable?

The 1-intersection graph G of T = (V, E) is a graph where E (the set of triples) is represented by the vertices of G and there is an edge between two vertices of G if and only if the corresponding triples in E intersect in exactly one element of V.

The aim of the project is to prove (or find counterexample to) the following statement.

If the 1-intersection graph of T = (V, E) is 2-colorable (bipartite graph) then the triple system T = (V, E) is also 2-colorable.

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