The \((k\text{-color})\ Ramsey\ number \(R(G_1, G_2, \ldots, G_k)\) of a sequence of graphs \(G_i\) is the smallest integer \(n\) for which the following is true: in any coloring of the edges of the complete graph \(K_n\) with colors \(1, 2, \ldots, k\), for some \(i \in \{1, 2, \ldots, k\}\) there is a copy of \(G_i\) whose edges are all colored with color \(i\) (a monochromatic copy of \(G_i\)).

Let \(S_t\) denote the star, the graph with \(t\) edges pairwise intersecting in the same vertex and let \(M_t\) be the matching (stripes), the graph with \(t\) pairwise disjoint edges.

**Exercise 1.** \((\text{special case of Cockayne, Lorimer, 1975})\) \(R(M_t, M_t) = 3t - 1\)

**Exercise 2.** \((\text{Burr and Roberts, 1973})\) Assume that \(m_1, \ldots, m_k\) are positive integers and \(Z = \sum_{i=1}^{k} (m_i - 1)\). Prove that

\[
R(S_{m_1}, S_{m_2}, \ldots, S_{m_k}) = \begin{cases} 
Z + 1 & \text{if } Z \text{ is even and some } m_i \text{ is even} \\
Z + 2 & \text{otherwise}
\end{cases}
\]

The chromatic number of a graph \(G\), denoted by \(\chi(G)\), is the minimum number \(m\) for which one can assign numbers from \(\{1, 2, \ldots, m\}\) to the vertices of \(G\) so that no two adjacent vertices gets the same number. The chromatic Ramsey number, \(\chi(G_1, G_2, \ldots, G_k)\) of a sequence of acyclic graphs \(G_i\) is the smallest integer \(n\) for which the following is true: in any coloring of the edges of any \(n\)-chromatic graph \(G\) with colors \(1, 2, \ldots, k\), for some \(i \in \{1, 2, \ldots, k\}\) there is a copy of \(G_i\) whose edges are all colored with color \(i\) (a monochromatic copy of \(G_i\)).

**Exercise 3.** \(\chi(G_1, G_2, \ldots, G_k) \geq R(G_1, G_2, \ldots, G_k)\).

We call a sequence \(G_1, G_2, \ldots, G_k\) of graphs \(k\)-good if there is equality in Exercise 3. It is known that stripes are \(k\)-good for every \(k\).

**Exercise 4.** \(\chi(M_t, M_t) = 3t - 1\) \(i.e.\) stripes are \(2\)-good.

The aim of the project is to prove or disprove some of the following statements.

- any sequence of stars is good \(\text{extension of Exercise 2}\)
- any sequence of stars and one stripe is good \(\text{would extend another result of Cockayne - Lorimer, 1975}\)
- the sequence of one star and two stripes is good \(\text{would extend a result of Gyárfás and Sárközy, 2012}\)

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