Stars and stripes

Research Project, 2015 Fall

The (k-color) Ramsey number $R(G_1, G_2, \ldots, G_k)$ of a sequence of graphs G_i is the smallest integer n for which the following is true: in any coloring of the edges of the complete graph K_n with colors $1, 2, \ldots, k$, for some $i \in \{1, 2, \ldots, k\}$ there is a copy of G_i whose edges are all colored with color i (a monochromatic copy of G_i).

Let S_t denote the *star*, the graph with t edges pairwise intersecting in the same vertex and let M_t be the *matching (stripes)*, the graph with t pairwise disjoint edges.

Exercise 1. *(special case of Cockayne, Lorimer, 1975)* $R(M_t, M_t) = 3t - 1$

Exercise 2. *(Burr and Roberts, 1973) Assume that* $m_1, \ldots, m_k$ *are positive integers and* $Z = \sum_{i=1}^{k}(m_i - 1)$. *Prove that*

$$R(S_{m_1}, S_{m_2}, \ldots, S_{m_k}) = \begin{cases} Z + 1 & \text{if } Z \text{ is even and some } m_i \text{ is even} \\ Z + 2 & \text{otherwise} \end{cases}$$

The chromatic number of a graph G, denoted by $\chi(G)$, is the minimum number m for which one can assign numbers from $\{1, 2, \ldots, m\}$ to the vertices of G so that no two adjacent vertices gets the same number. The *chromatic Ramsey number*, $\chi(G_1, G_2, \ldots, G_k)$ of a sequence of acyclic graphs G_i is the smallest integer n for which the following is true: in any coloring of the edges of any n-chromatic graph G with colors $1, 2, \ldots, k$, for some $i \in \{1, 2, \ldots, k\}$ there is a copy of G_i whose edges are all colored with color i (a monochromatic copy of G_i).

Exercise 3. $\chi(G_1, G_2, \ldots, G_k) \geq R(G_1, G_2, \ldots, G_k)$.

We call a sequence $G_1, G_2, \ldots, G_k$ of graphs k-good if there is equality in Exercise 3. It is known that stripes are k-good for every k.

Exercise 4. $\chi(M_t, M_t) = 3t - 1$ *i.e. stripes are* 2*-good.*

The aim of the project is to prove or disprove some of the following statements.

- any sequence of stars is good (extension of Exercise 2)
- any sequence of stars and one stripe is good (would extend another result of Cockayne - Lorimer, 1975)
- the sequence of one star and two stripes is good (would extend a result of Gyárfás and Sárközy, 2012)

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