The Ramsey number of a bow-tie

What is a bow-tie? Geometrically it is two triangles with a common vertex. In graph theory, it is a graph with six edges $12, 23, 13, 34, 45, 35$. However, in hypergraph theory it is simply two triples $123, 345$ and *this bow-tie is the subject of the research.*

An unpublished note of Paul Erdős and Vera Sós states the following.

Theorem 1. *At most n triples can be selected from a set of n elements so that no two of them form a bow-tie.*

Exercise 1. Prove Theorem 1.
Exercise 2. Show that Theorem 1 is best possible for infinitely many values of n, i.e. n triples can be selected from n elements so that no two form a bow-tie.
Exercise 3. State (and prove if you can) a form of Theorem 1 that is best possible for every n.

Definition 1. *The k-color Ramsey number of a bow-tie, b_k, is the smallest n with the following property: if the triples of an n-element set are colored with k colors, there is a bow-tie with both triples having the same color.*

Exercise 3. Prove that $b_2 = 5, b_3 = b_4 = b_5 = 6$.
Exercise 4. Prove that $b_8 = 9$.

The aim of the project is to find missing values of b_k, in particular improve the following.

Proposition 1. $9 \leq b_{11} \leq b_{12} \leq b_{13} \leq b_{14} \leq 10$.