Monochromatic connected pieces

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Prove propositions you like, it helps to decide whether you are interested in (qualified for) the subject "Monochromatic connected pieces" (offered within BSM Elective Undergraduate Research Program, 2013 SPRING).

Proposition 1 *(Warm up) In every 2-coloring of the edges of a complete graph there is a monochromatic spanning tree.*

Proposition 2 *In every 2-coloring of the edges of a complete graph there is a monochromatic spanning tree of height at most two.*

Proposition 3 *In every 2-coloring of the edges of a complete graph there is a monochromatic spanning octopus (disjoint paths apart from a common root).*

Proposition 4 *In every 2-coloring of the edges of a complete graph there is a monochromatic spanning subgraph with diameter at most three.*

Proposition 5 *In every 2-coloring of the edges of a k-chromatic graph, there is a monochromatic tree with at least k vertices.*

Proposition 6 *The vertices of a complete graph form a convex polygon on a plane. Prove that in every 2-coloring of the edges there is a monochromatic spanning tree without crossing edges.*

Proposition 7 *Suppose that the edges of a complete graph are colored so that no triangle is colored with three distinct colors. Prove that there is a monochromatic spanning tree.*

Proposition 8 *In every 2-coloring of the edges of a complete graph $K_n, n \geq 5$, there is a monochromatic 2-connected subgraph with at least $n-2$ vertices.*

Let $\alpha(G)$ denote the maximum number of pairwise nonadjacent vertices of a graph.

Proposition 9 *In every 2-coloring of the edges of a graph G there is a monochromatic connected subgraph with at least $\frac{|V(G)|}{\alpha(G)}$ vertices.*

Proposition 10 *In every 2-coloring of the edges of a graph of minimum degree $\delta(G) \geq \frac{3|V(G)}{4}$ there is a monochromatic component with more than $\delta(G)$ vertices.*

Proposition 11 *In every 3-coloring of the edges of the complete 3-uniform hypergraph K_n^3 there is a monochromatic spanning component.*

Proposition 12 *Suppose that the edges of the complete 3-uniform hypergraph K_3^n are 4-colored so that there is no tetrahedron (K_4^3) whose edges are colored with four distinct colors. Then there is a monochromatic spanning component.*