Beating the Delsarte bound – Exercises

(1) Let $p = 17$. List the quadratic residues modulo p, and find the maximal number of elements mod p such that no two of them differ by a quadratic residue.

(2) Let p be a prime. Prove that the maximal number of elements mod p such that no two of them differ by a quadratic residue is less than $\sqrt{p}$. (There is an elementary combinatorial argument that you can find.)

(3) Let $A(n, d)$ denote the maximal number of 0-1 sequences of length n such that any two of them differ at least in d bits. Prove that $A(n, d) \leq 2A(n-1, d)$, and $A(n-1, 2d-1) = A(n, 2d)$.

(4) Determine $A(7, 4)$.