Monochromatic connected pieces

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Prove propositions you like, it helps to decide whether you are interested in (qualified for) the subject "Monochromatic connected pieces" (offered within BSM Elective Undergraduate Research Program, 2013 SPRING). Further material is in my survey paper 2011/139, http://www.renyi.hu/ gyarfas/.

Proposition 1 *(Warm up) In every 2-coloring of the edges of a complete graph there is a monochromatic spanning tree.*

Proposition 2 *In every 2-coloring of the edges of a complete graph there is a monochromatic spanning tree of height at most two.*

Proposition 3 *In every 2-coloring of the edges of a complete graph there is a monochromatic spanning octopus (disjoint paths apart from a common root).*

Proposition 4 *In every 2-coloring of the edges of a complete graph there is a monochromatic spanning subgraph with diameter at most three.*

The next one is from the 2012 Schweitzer competition (take home problems for a week to Hungarian University students)

Proposition 5 *In every 2-coloring of the edges of a k-chromatic graph, there is a monochromatic tree with at least k vertices.*

Proposition 6 *The vertices of a complete graph form a convex polygon on a plane. Prove that in every 2-coloring of the edges there is a monochromatic spanning tree without crossing edges.*

Proposition 7 *Suppose that the edges of a complete graph are colored so that no triangle is colored with three distinct colors. Prove that there is a monochromatic spanning tree.*

Proposition 8 *In every 2-coloring of the edges of a complete graph $K_n, n \geq 5$, there is a monochromatic 2-connected subgraph with at least $n - 2$ vertices.*

Let $\alpha(G)$ denote the maximum number of pairwise nonadjacent vertices of a graph.

Proposition 9 *In every 2-coloring of the edges of a graph G there is a monochromatic connected subgraph with at least $\frac{|V(G)|}{\alpha(G)}$ vertices.*

Proposition 10 *In every 2-coloring of the edges of a graph of minimum degree $\delta(G) \geq \frac{3|V(G)}{4}$ there is a monochromatic component with more than $\delta(G)$ vertices.*

Proposition 11 *In every 3-coloring of the edges of the complete 3-uniform hypergraph K_n^3 there is a monochromatic spanning component.*

Proposition 12 *Suppose that the edges of the complete 3-uniform hypergraph K_3^n are 4-colored so that there is no tetrahedron (K_4^3) whose edges are colored with four distinct colors. Then there is a monochromatic spanning component.*