**Conjecture and Proof    C&P**

*Instructor:* Dr. Róbert FREUD

*Text:* M. Laczkovich, Conjecture and Proof + handouts

*Prerequisite:* Introductory math courses

The course intends to show the many and often surprising interrelations between the various branches of mathematics (algebra, analysis, combinatorics, geometry, number theory, and set theory), to give an introduction to some basic methods of proofs via the active problem solving of the students, and also to exhibit several unexpected mathematical phenomena. Among others we plan to find answers to the following questions:

- Can the real function $f(x) = x$ be written as the sum of (finitely many) periodic functions?
- Can you chop a cube by plane cuts into finitely many polyhedra and reassemble a regular tetrahedron (using each part once)?
- Can you "cut" a sphere into finitely many (not necessarily "nice") subsets and "reassemble" two(!) copies of the original sphere?
- Can you color the numbers $1, 2, \ldots, 2012$ using two colors so that for no $a$ and $d > 0$ holds that $a, a+d, a+2d, \ldots, a+17d$ all have the same color?
- Can every integer large enough be represented as the sum of 6 sixth powers, i.e. is $n = x_1^6 + \ldots + x_6^6$ solvable in nonnegative integers $x_i$ if $n$ is large enough?

*Topics:*

Irrational, algebraic and transcendental numbers, their relations to approximation by rationals and to cardinalities.

Pigeonhole principle, counting arguments, invariants for proving impossibility, applications in combinatorics and number theory.

Vector spaces and fields, field extensions, geometric constructions (doubling the cube, squaring the circle, trisecting the angle, constructing regular polygons), Hamel bases, Cauchy's functional equation, Sidon-sets.

Isometries, geometric and paradoxical decompositions: Bolyai–Gerwien theorem, Hilbert's third problem, Hausdorff paradox, Banach–Tarski paradox.