Classical Algebra: Spring 2011

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Introduction:

The purpose of this course is to explore some fundamental topics in mathematics which you may or may not have some familiarity. The topics are concepts which every mathematician should be aware of but are particularly useful in the study of advanced areas of abstract algebra. If you have had a first class in abstract algebra you are probably familiar with some of the ideas we present here but maybe not to the degree of understanding that we would like BSM students to have. If you have had a class in Galois Theory you have probably mastered the concepts of this class . Of course, in that case you are welcome to sit in but you may find that it is not the best use of your time.

It is our hope that your interest in and love of mathematics will be the main incentive for attending this class. This class carries no credit but if you attend 5 of the 6 sessions a note will appear on your transcript that you attended. The approach will be through problem solving so each session will involve considering problems in class. You will also receive problems to consider outside of class that we will discuss at the next session.

A Rough Syllabus:

During the first week we will consider basic properties of the complex numbers. This includes learning how to solve equations involving complex numbers, in particular equations of the form $x^n = 1$ and studying geometric properties of complex numbers. Specifically we will consider algebraic and trigonometric forms , conjugates, length , norms, De Moivre's Theorem , nth roots, roots of unity, primitive roots, and geometric, algebraic and combinatorial applications. Typical problems might include:

1) Evaluate $(cos(\frac{\pi}{6}) + isin(\frac{\pi}{3}))^8$

2) Show that the two triangles ABC and DEF are similar iff the determinant of $\begin{pmatrix} A & D & 1 \\ B & E & 1 \\ C & E & 1 \end{pmatrix} = 0.$

3) Evaluate $\Sigma\binom{n}{4k}$ where $\binom{n}{k}$ represents a binomial coefficient.

During the second and third weeks we will focus on properties of polynomials. Specifically we will consider divisibility, the division algorithm, the Euclidean Algorithm, polynomials over $\mathbf{Z}_p$, Fermat's Theorem , Wilson's Theorem, irreducibility over $\mathbf{Z}$ and $\mathbf{Z}_p$, algebraic closure, connection between roots and coefficients, symmetric functions , cyclotomic polynomials, and inequalities related to the AGM inequality. Typical problems might include:

1) Show that $x^2 + x + 1$ divides $x^{3k} + x^{3n+1} + x^{3m+2}$ for all integers k, n, m.

2) Find all monic irreducible quadratics in $\mathbf{Z}_7$.

3) Find a cubic equation whose roots are the squares of the roots of $x^3 - x^2 + 3x - 10$.

4) Prove that $x + x^{-1} \geq 2$ for all $x > 0$.

5) Suppose $x + y + z = a$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{a}$ Show that x, y or z equals a.

It is my hope that almost all BSM students will at least attend the first session of this class to determine if it is a class which may be of value to them.