

## Classical Algebra Workshop

February 2009

### Week 1: Complex Numbers

**First Meeting:** Algebraic and trigonometric forms, conjugates, length, norm, operations, De Moivre's Theorem,  $n^{\text{th}}$  roots, roots of unity, primitive roots, geometric, algebraic and combinatorial applications.

**Second Meeting:** Problem Solving Session

Sample exercises and problems:

- 1) Find  $(\cos(30^\circ) + i\sin(60^\circ))^8$
- 2) Evaluate the sum,  $\sum_k \binom{n}{3k}$
- 3) Find the sum, product and sum of the squares of the  $n^{\text{th}}$  roots of unity.

### Week 2: Polynomials I

**First Meeting:** Well Ordering Principal, Division Algorithm, divisibility, Euclidean Algorithm, G.C.D, unique factorization, polynomials over  $\mathbf{Z}_p$ , applications to number theory, Fermat's Theorem, Wilson's Theorem, irreducibility in  $\mathbf{Z}_p$ , factor theorem, remainder theorem.

**Second Meeting:** Problem Solving Session

Sample exercises and problems:

- 1) Find the GCD of  $x^3 + 2x^2 + 3x + 2$  and  $x^4 + x^3 - x - 1$ . If  $D(x)$  is the GCD find polynomials  $s(x)$  and  $t(x)$  so that  $D(x) = (x^3 + 2x^2 + 3x + 2)s(x) + (x^4 + x^3 - x - 1)t(x)$ .
- 2) Show that  $x^2 + x + 1$  divides  $x^{3k} + x^{3n+1} + x^{3m+2}$
- 3) Find all the irreducible monic quadratics in  $\mathbf{Z}_7[x]$

### Week 3: Polynomials II

**First meeting:** Fundamental Theorem of Algebra, algebraic closure, integer and rational root tests, connection between roots and coefficients, multiple roots, symmetric functions, symmetric function theorem, Gauss's Lemma, Eisenstein Criterion, cyclotomic polynomials, Newton's Formula, AGM type inequalities.

**Second Meeting:** Problem Solving Session

Sample exercises and problems:

1. Find a cubic equation whose roots are the squares of the roots of  $x^3 - x^2 + 3x - 10$ .
2. Prove that  $x + x^{-1} \geq 2$  for all  $x > 0$ .
3. Suppose  $x + y + z = a$  and  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{a}$  Show that one of  $x, y, z$  must equal  $a$