Classical Algebra Workshop

September 2008

Week 1: Complex Numbers

First Meeting: Algebraic and trigonometric forms, conjugates, length, norm, operations, De Moivre's Theorem, nth roots, roots of unity, primitive roots, geometric, algebraic and combinatorial applications.

Second Meeting: Problem Solving Session

Sample exercises and problems:

1) Find $(cos(30°) + isin(60°))^8$

2) Evaluate the sum, $\Sigma_k \binom{n}{3k}$

3)Find the sum, product and sum of the squares of the n-th roots of unity.

Week 2: Polynomials I

First Meeting: Well Ordering Principal, Division Algorithm, divisibility, Euclidean Algorithm, G.C.D, unique factorization, polynomials over$\mathbf{Z}_p$, applications to number theory, Fermat's Theorem, Wilson.s Theorem, irreducibility in $\mathbf{Z}_p$, factor theorem, remainder theorem.

Second Meeting: Problem Solving Session

Sample exercises and problems:

1) Find the GCD of $x^3 + 2x^2 + 3x + 2$ and $x^4 + x^3 - x - 1$. If D(x) is the GCD find polynomials s(x) and t(x) so that $D(x) = (x^3 + 2x^2 + 3x + 2)s(x) + (x^4 + x^3 - x - 1)t(x)$.

2) Show that $x^2 + x + 1$ divides $x^{3k} + x^{3n+1} + x^{3m+2}$

3) Find all the irreducible monic quadratics in $\mathbf{Z_7[x]}$

Week 3: Polynomials II

First meeting: Fundamental Theorem of Algebra, algebraic closure, integer and rational root tests, connection between roots and coefficients, multiple roots, symmetric functions, symmetric function theorem, Gauss's Lemma, Eisenstein Criterion, cyclotomic polynomials , Newton's Formula, AGM type inequalities.

Second Meeting: Problem Solving Session

Sample exercises and problems:

1. Find a cubic equation whose roots are the squares of the roots of $x^3 - x^2 + 3x - 10$.

2. Prove that $x + x^{-1} \geq 2$ for all $x > 0$.

3. Suppose $x + y + z = a$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{a}$ Show that one of x, y, z must equal a