Abstracts

Marianna Bolla, University of Technology and Economics, Budapest

The role of the modularity matrices in the discrepancy based spectral clustering

Since the 1970s, classical spectral clustering algorithms intensively use the adjacency or Laplacian matrices of a graph to give an approximate solution for minimum or maximum multiway cut problems. The so-called spectral relaxation applies to the top or bottom eigenvalues together with the corresponding eigenvectors. At the turn of the millennium, social network scientists introduced the modularity matrix, the entries of which measure the difference between the actual and expected connectedness of the vertices, under independent attachment of them. The modularity matrix and its normalized version is capable to estimate the so-called Newman–Girvan modularity, see [1]. The balance between the positive and negative eigenvalues decides whether the network has a community or anticommunity structure, and we can also estimate the between- or within-cluster (module) discrepancies with the structural eigenvalues (of large absolute value). Above the connection to the expander mixing lemma, we will characterize graphs with no positive modularity eigenvalue (i.e., no community structure) at all, see [2] (the middle authors are former BSM students).

References

[1] Bolla, M., Penalized versions of the Newman–Girvan modularity and their relation to normalized cuts and k-means clustering, *Phys. Rev. E* **84** (2011), 016108.

[2] Bolla, M., Bullins, B., Chaturapruek, S., Chen, S., Friedl, K., Spectral properties of modularity matrices, *Linear Algebra and Its Applications* **73** (2015), 359-376.

András Gyárfás, Rényi Institute, Budapest

Open problems from my research experience courses

- **Staircases.** 2012 Fall, Siyuan Cai, Gillian Grindstaff

  A 0-staircase in an $n \times n$ $0-1$ matrix is a sequence of zeroes which goes right in rows and down in columns, possibly skipping elements, but zero at each turning point. A 1-staircase is defined similarly. *Do we have a* 0*-staircase or a* 1*-staircase having* $n-1$ *entries in every* $n \times n$ $0-1$ *matrix?*

- **Good graph hunting.** 2014 Fall, Philip Garrison

  Assume that a graph is the union of three P_4*-free graphs. Is it true that the chromatic number of* G *is at most 5?*

- **Chromatic Ramsey number of hypergraphs.** 2015 Spring, Melissa U. Shermann-Bennett, Alex W.N.Raisanovsky

  Let H be a system of 4-element subsets of a set. The 1-*intersection graph* of H represents the sets by vertices and edges are defined if the corresponding sets intersect in exactly one element. *Assume that the* 1*-intersection graph of* H *is a bipartite graph. Is it true that* H *is also bipartite (has a vertex partition into two parts such that each* 4*-set intersects both parts)?*

- **Ramsey Theory on Steiner triple systems.** 2016 Summer, Elliot Granath, Jerry Hardee, Trent Watson, Xiaoze Wu

  The *sail* is the configuration of four triples on seven points, three pairwise intersecting in the same point p and the fourth intersecting all of them in points different from p. *Is it true that in every red-blue coloring of the triples of any sufficiently large (perhaps* $n \geq 13$*) Steiner triple system, there is a red or a blue sail?*

I will discuss some results as well, see http://www.bsmath.hu/articles.html. Best bound on staircases is in Csányi, Hajnal, Nagy, Electronic Journal of Combinatorics, vol. 23 (2016) P2.17

Kiran Kedlaya, University of California, San Diego

A star chart for number theory: the L-Functions and Modular Forms Database

The L-Functions and Modular Forms Database (LMFDB) is a web site, developed by an international collaboration, that aims to build a catalog of objects in number theory and highlight both known and conjectural links among these objects. I'll give a quick tour of the site and some of its features, includings its use of "knowls" to provide context-free descriptions of key terms and objects. No background in number theory will be assumed.

Ryan Matzke, University of Minnesota – Twin Cities

On Fejes-Tóth's conjectures on the sums of angles

In 1959, Fejes-Tóth posed two conjectures about optimal point distributions on the sphere, more precisely, about the maximizers of the following discrete energies: (a) the pairwise sum of angles (geodesic distances) between N unit vectors in S^d, (b) the sum of non-obtuse angles, i.e. the sum of angles between N lines defined by vectors in S^d. Continuous versions of these conjectures can also be stated. While the questions are clearly related, their nature is rather different: the first case induces strong "repulsion", while the second imposes orthogonality. Through a new analogue of the Stolarsky Invariance Principle from discrepancy theory, we characterize the maximizers of the sum of angles, verifying the first conjecture. We also provide improved energy bounds for the sum of line angles by relating this problem to the so-called frame potential. In addition, we discuss several new solutions to the only settled case of the second conjecture for $d = 1$.

Dezső Miklós, Rényi Institute, Budapest

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Elizabeth Milićević, Haverford College

Projections and bijections on alcoves

Affine Weyl groups and their parabolic quotients are used extensively as indexing sets for objects in representation theory, algebraic geometry, and number theory. We can realize the elements of certain quotients via intuitive geometric and combinatorial models such as abacus diagrams, alcoves, root lattice points, and core partitions. Berg, Jones, and Vazirani describe a bijection between n-cores with first part equal to k and $(n-1)$-cores with first part less than or equal to k. In this talk, we discuss how to generalize this bijection to other classical Lie types using a geometric visualization called the affine hyperplane arrangement. From this geometry, we obtain a lovely bijection and associated combinatorics!

Cory Palmer, University of Montana
Deranged matchings

The number of derangements of an n-element set can be realized as the number of perfect matchings in a complete bipartite graph $K_{n,n}$ with a perfect matching removed. For large n, this value is approximately $n!/e$. A related problem is the number of perfect matchings in the complete graph K_{2n} with a perfect matching removed. For large n, this value is approximately $(2n-1)!!/\sqrt{e}$. In this talk we discuss a common generalization of these parameters by investigating the number of perfect matchings in certain k-partite graphs.

Sam Spiro, University of California, San Diego
Forbidden configurations and forbidden families

For a graph G, the extremal number $ex(n, G)$ is defined to be the maximum number of edges an n vertex graph can have without containing G as a subgraph. There exists a natural extension of this function to the problem of avoiding hypergraphs.

We will say that a matrix A is simple if it is a (0,1)-matrix with no repeated columns (i.e. it is the incidence matrix of a simple hypergraph). Given a (0,1)-matrix F, we say that a matrix A has F as a configuration, denoted $F \prec A$, if there is a submatrix of A which is a row and column permutation of F. We define $\text{forb}(m, F)$ to be the maximum number of columns an m-rowed simple matrix that does not contain F as a configuration can have. More generally, we define $\text{forb}(m, \mathcal{F})$ for $\mathcal{F}$ a set of matrices to be the maximum number of columns an m-rowed simple matrix that avoids every matrix of $\mathcal{F}$ can have.

We compute asymptotic values for $\text{forb}(m, \{F, G\})$ when F and G are minimal quadratic or minimal cubic configurations. This is joint work with Attila Sali, and the research was conducted through the BSM Research Opportunities course.

Rebecca Swanson, Colorado School of Mines
Flipped learning in mathematics

TBA

Russ Woodroofe, University of Primorska
Invariable generation of alternating groups by prime-power elements

Two elements x, y *invariably generate* a group G if any conjugate x together with any conjugate of y generates G. Invariable generation of a group G by prime or prime-power elements has consequences for fixed-point-free actions on certain geometries with G actions. In previous work, John Shareshian and I have shown that, assuming the Riemann hypothesis, the alternating groups A_n are invariably generated by elements of prime order for all n except for n on a set of asymptotic density 0. On the other hand, we have constructed infinitely many examples that are not invariably generated by such elements. Prime divisibility of binomial coefficients turns out to have a strong relationship with this problem! I'll give an overview of the project, and discuss ongoing work with Bob Guralnick and Shareshian, where we show that many alternating groups are invariably generated by two elements of prime-power order.