Machine Learning in Combinatorial Geometry

BSM Research opportunities project

Prerequisites: strong command of the python numerical libraries (numpy, scipy). linear programming. familiarity with machine learning frameworks (scikit-learn, pytorch) or fast CPU algorithm programming (C++, Rust, numba) is an advantage.

What: We attempt to make progress in some problems in combinatorial geometry.
How: Our approach is to phrase these problems as search problems, and use computer search to solve them. The plan is to use advanced machine learning methods such as deep reinforcement learning where necessary, and use classic search methods when they already get the job done.
Why: We see a comparative advantage in attacking some classic pure math problems with computer search. Most of the mathematicians who looked into such problems during the years haven't employed computer methods, and certainly not modern machine learning techniques.

By a conjecture of Paul Erdős, the supremum $m_1(\mathbf{R}^2)$ of the densities of the measurable planar sets avoiding unit distances cannot exceed ¼. (A set avoids unit distances if no two elements of it are one unit apart.) Please check out our recent paper The density of planar sets avoiding unit distances in which we convert this classic geometry question into a linear programming formulation that we then solve using computer search. (You can start with our talk at the Geometry Seminar for an overview of the proof.)

There is a lot of opportunity for related work here:

- Another question about unit distance graphs on the plane (also from Erdős) is about the maximum possible density of n-vertex unit distance graphs on the plane, see this Wikipedia paragraph. Tree search methods or reinforcement learning could aid in finding optimal configurations for small values of n. (See Ágoston and Pálvölgyi 2020 Table 1 for the current state of the art.)
- In Roucairol and Cazenave, 2022, a number of Spectral Graph Theory conjectures are solved using Monte Carlo Search, and it is pointed out that more such problems could be solved similarly.
- We are still working on getting an upper bound on $m_1(\mathbf{R}^2)$ closer to the best known lower bound 0.2293 given by the Croft construction. You could help us in this endeavor, for example in figuring out which equations, if any, can be thrown out from our LPs, by investigating the dual coefficients.

- A prominent related problem (the measurable chromatic number of the plane) might be attacked with a variant of our approach. You could partake in this quest too: For example, we'd be very glad if you could adapt the verification procedure that is described in Section 8 of our paper to this setting.
- We are open to considering other problems: If you have an idea about another problem that may be approached by computer search, tell us about it!

Qualifying problem:

Write a Python function `get_children(vertices: ndarray) -> ndarray` that given a nonempty finite planar subset X represented as a 1D complex array, outputs a 2D complex array, where the rows represent all the planar subsets $X'=X\cup\{x'\}$ where $x'\in\mathbf{C}$ is a planar point such that

- it is of unit distance to at least two points of X up to numerical precision and
- it is not closer than 0.1 to any point of X up to numerical precision.

(In the tree described on page 14 of our paper [The density of planar sets avoiding unit distances](), these are the children of the node given by X.)

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