Quick introduction
Note: Figures 1-4 can be found on page 2.

1. A *knot* K is a smooth embedding of the circle into $\mathbb{R}^3$, that is a map $f\colon S^1 \to \mathbb{R}^3$ which is injective (here $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$). Two knots K_0 and K_1 are *isotopic* if there is a smooth map $F\colon S^1 \times [0, 1] \to \mathbb{R}^3$ such that (1) $F_0 = F|_{S^1\times\{0\}}\colon S^1 \to \mathbb{R}^3$ is the map defining K_0 and $F_1 = F|_{S^1\times\{1\}}\colon S^1 \to \mathbb{R}^3$ is the map defining K_1, and (2) all $F_t = F|_{S^1\times\{t\}}\colon S^1 \to \mathbb{R}^3$ is a map defining a knot K_t (here $t \in [0, 1]$). So "K_0 can be moved smoothly to K_1. This definition gives an equivalence relation on knots, a class is called a *knot type*, and from now on we will study knot types.
2. A knot can be projected down to a plane, and choosing the plane generically, the projecton will be also an embedding (now as a map $S^1 \to \mathbb{R}^2$) away from finitely many points, where we have a *double point*. In those points, in order to recover the knottidness of K, we use the convention to denote the strand underpassing by an interrupted line segment. An example is shown by Figure 1. The result is called a knot diagram.
3. A knot has many such diagrams; the *crossing number* of the knot is the minimal number of double points, when considering all diagrams for K. There is a single knot with crossing number zero, called the unknot. This has the diagram of a round circle.
4. A *three-coloring* of a diagram is simply the coloring of each uninterrupted segment with a color out of red, white and green, satisfying two rules: (1) at least two colors are used, and (2) at a crossing either all three colors are used, or only one. (So there is no crossing where the meeting three strands are colored with two colors.)
5. The diagram gives rise to a graph (the *black graph*) as follows: checkerboard color the domains of the diagram (meaning that each domain of the complement is either black or white, and two black domains cannot share an edge, and the same for two white domains). So same color domains either do not meet at all, or they meet at a crossing (where occupy to opposing quadrants). See Figure 2. Now define the black graph Γ_B of the diagramby taking the black domains as vertices, and connect two black domains through the crossings shared by the two domains. (In a similar manner we can define the white graph.)

Problems:

- Show that there are no knots with crossing number one and two.
- Show that if the black graph of a knot is a tree, then the knot is the unknot.
- Show that the trefoil knot (the knot of Figure 3) is three-colorable, while the Figure-8 knot (the knot shown by Figure 4) is not.

FIGURE 1:

FIGURE 2:

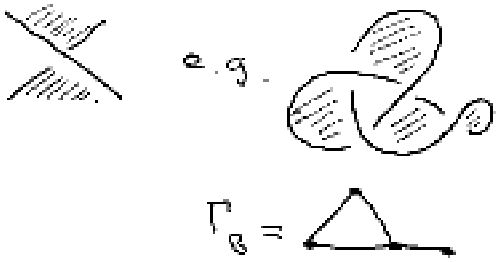

FIGURE 3:

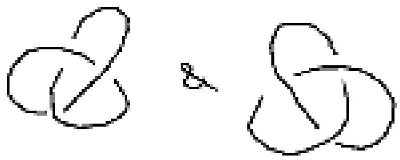

FIGURE 4: