**Finite Geometries**

Text: handouts

Prerequisite: basic linear algebra, finite fields, basic classical projective geometry

Topics:

- Axiomatic treatment of projective and affine planes (examples of finite planes, basic combinatorial properties, the existence problem, Bruck-Ryser Theorem).
- Collineations
  (point and line orbits, central-axial collineations, Theorem of Baer, translation planes).
- Coordinatization
  (the method of M. Hall, ternary rings, quasifields, semifields, nearfields and the corresponding planes).
- Arcs, ovals and hyperovals
  (examples, Theorem of Bose, complete arcs, Theorem of Segre).
- Blocking sets
  (examples, combinatorial bounds on the size, Bruen-Pelikán Theorem, lacunary polynomials, Theorem of Blokhuis, blocking sets of Rédei type).
- $(k, n)$-arcs and multiple blocking sets
  (estimates on the size, nuclei, Segre-Korchmáros Lemma, maximal arcs, the construction of Denniston).
- Complete arcs
  (linear systems, the generalization of Menelaus Theorem, Lemma of Tangents, upper bounds on the size).
- Higher dimensional spaces
  (axiomatic treatment, vector space model, homogeneous coordinates, combinatorial properties, projectivities, subspaces).

- Polarities in $\mathrm{PG}(n, q)$
  (classification, ordinary-, Hermitian-, null- and pseudo polarities, self-conjugate points and subspaces).
- Quadratic surfaces, Hermitian varieties
  (canonical forms, classification, basic geometric and combinatorial properties).
- Generalized quadrangles, Möbius planes
  (axiomatic treatment, examples, classical GQ-s, egglike planes).
- Arcs and caps in higher dimensional spaces
  (ovoids in $\mathrm{PG}(3, q)$, estimates on the sizes of complete caps and arcs).
- Higher dimensional representations
  (spreads, Plücker coordinates, Klein correspondence, spreads, translation planes).
- Applications to graph theory and extremal combinatorics
  (Turán type problems, one-factorization, Moore graphs and cages, linear spaces, De Bruijn-Erdőss Theorem, Zarankiewicz problem).
- Applications to coding theory and cryptography
  (Hamming codes, MDS codes and arcs, geometric constructions of secret sharing schemes).