KOMPLEX SZÁMOK

PRACTICE PROBLEMS ABOUT COMPLEX NUMBERS

1. Solve the following equations over the complex numbers
(1)a)$x^2 + 1 = 0$ b) $x^2 = -12$ c) $x^2 + 2x + 2 = 0$ d) $x^2 + 2ix + 1 = 0$
(2)a) $x^6 = 1 + i$ b) $x^4 + 4 = 0$ c) $x^n = -1$ d)$x^{12} = -32 + 32\sqrt{3}i$

2. Where are the following sets of numbers on the complex plane?
   a)$\{z \mid Re(z+1) > Im(z-3-i)\}$ b)$\{z \mid |z-i-2| \le 2\}$
   c)$\{z \mid Re(z-3i) = |z-2+i|\}$ d)$\{z \mid 1/z = \bar{z}\}$
   e)$\{z \mid Re(z-1)/(z+1) = 0\}$ f)$\{z \mid Im(z-1)/(z+1) = 0\}$

3. Find the order of the following numbers:
   $1, -1, -i, 1+i, (1+i)/\sqrt{2}, \cos(\sqrt{2}\pi) + i\sin(\sqrt{2}\pi), \cos 336° + i\sin 33°$

4. Show that $Re(1/z) > 0 \iff Re(z) > 0$.

5. Mely komplex számokra igaz, hogy a) $\bar{z} = z^2$; b) $\bar{z} = z^n$?

6. Igazold, hogy $\left(\dfrac{1 + itg\alpha}{1 - itg\alpha}\right)^n = \dfrac{1 + itgn\alpha}{1 - itgn\alpha}$.

7. Mely komplex számokra igaz, hogy $|z| + z = 2 + i$?

8. Find the 24th roots of unity and their orders.

9. Find the sum, product and the sum of the squares of the n-th roots of unity.

10. Multiply the 6-th roots of unity by the 4-th roots of unity. What do we obtain?

11. Find the algebraic form of $\sqrt{a + bi}$

12. Compute the following powers (hint:first find the trigonometric form): i^n, $(\frac{1}{2} + \frac{\sqrt{3}i}{2})^{2003}$, $(-\frac{1}{2} + \frac{\sqrt{3}i}{2})^{2003}$, $(\frac{1}{2}i + \frac{\sqrt{3}}{2})^{2003}$, $(\sqrt{3}i + 1)^n$, $(i + \sqrt{3})^n$

13. Let $x + \frac{1}{x} = -1$. Find $x^{65} + (\frac{1}{x})^{65}$
   Let $x + \frac{1}{x} = 2\cos\alpha$. Prove that $x^n + (\frac{1}{x})^n = 2\cos n\alpha$

14. Let $z \ne 1$ and $|z| = 1$. Prove that there is a unique $t \in R$ such that $z = (t+i)/(t-i)$.

15. Where are the following points on the complex plane: $\{\frac{1}{1-z} : |z| = 1\}$