Good Graph Hunting

Research Project

The (k-color) Ramsey number $R_k(G)$ of a graph G is the smallest integer n for which the following is true: in any coloring of the edges of the complete graph K_n with colors $1, 2, \ldots, k$, for some $i \in \{1, 2, \ldots, k\}$ there is a copy of G whose edges are all colored with color i (a monochromatic copy of G).

Exercise 1. Let P_3 denote the path on 3 vertices. Prove that

$$R_k(P_3) = \begin{cases} k+1 & \text{if } k \equiv 0 \pmod{2} \\ k+2 & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

The chromatic number of a graph G, denoted by $\chi(G)$, is the minimum number m of colors for which one can color the vertices of G with m colors so that no two adjacent vertices are colored with the same color. Exercise 1 is true in the following more general form:

Exercise 2. Suppose that $\chi(G) = R_k(P_3)$. Then, in every k-coloring of the edges of G there is a monochromatic P_3 .

We call a graph H k-good if there is a monochromatic copy of H in every k-coloring of the edges of any $R_k(H)$ -chromatic graph. A graph is good if it is k-good for every positive integer k. Exercise 2 states that the graph P_3 is good.

Exercise 3. A graph H is 1-good if and only if H has no cycle.

The aim of the project is to find further good (or 2-good) graphs.

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András Gyárfás