## Good Graph Hunting

Research Project

The ( $k$-color) Ramsey number $R_{k}(G)$ of a graph $G$ is the smallest integer $n$ for which the following is true: in any coloring of the edges of the complete graph $K_{n}$ with colors $1,2, \ldots, k$, for some $i \in\{1,2, \ldots, k\}$ there is a copy of $G$ whose edges are all colored with color $i$ (a monochromatic copy of $G$ ).

Exercise 1. Let $P_{3}$ denote the path on 3 vertices. Prove that

$$
R_{k}\left(P_{3}\right)=\left\{\begin{array}{lll}
k+1 & \text { if } k \equiv 0 & (\bmod 2) \\
k+2 & \text { if } n \equiv 1 & (\bmod 2)
\end{array}\right.
$$

The chromatic number of a graph $G$, denoted by $\chi(G)$, is the minimum number $m$ of colors for which one can color the vertices of $G$ with $m$ colors so that no two adjacent vertices are colored with the same color. Exercise 1 is true in the following more general form:

Exercise 2. Suppose that $\chi(G)=R_{k}\left(P_{3}\right)$. Then, in every $k$-coloring of the edges of $G$ there is a monochromatic $P_{3}$.

We call a graph $H k$-good if there is a monochromatic copy of $H$ in every $k$-coloring of the edges of any $R_{k}(H)$-chromatic graph. A graph is good if it is $k$-good for every positive integer $k$. Exercise 2 states that the graph $P_{3}$ is good.

Exercise 3. A graph $H$ is 1-good if and only if $H$ has no cycle.
The aim of the project is to find further good (or 2-good) graphs.

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