# Coloring $t$-intersecting hypergraphs

EUR proposal 2013 summer

András Gyárfás

June 6, 2013

A *proper coloring* of a hypergraph is a coloring of its vertices so that no edge is monochromatic, i.e. contains at least two vertices with distinct colors. It is well-known that intersecting hypergraphs without singleton edges have proper colorings with at most three colors. This statement is from the seminal paper of Erdős and Lovász. Recently Blais, Weinstein and Yoshida suggested a generalization in [1]. They consider $t$-intersecting hypergraphs, in which any two edges intersect in at least $t$ vertices and they call a coloring of the vertices $c$-strong if every edge is colored with at least $c$ distinct colors. One of the problems they consider is equivalent to the following.

**Problem 1.** *([1]) Suppose that every edge has more than $t$ vertices in a $t$-intersecting hypergraph $\mathcal{H}$. Is there a $(t+1)$-strong vertex coloring of $\mathcal{H}$ where the number of colors is bounded by a function of $t$? In particular, is there a $t+1$-strong vertex coloring with at most $2t+1$ colors? If true then best possible, shown by all $2t$-element subsets of a $3t$-element ground set.*

Notice that for $t = 1$ the answer to Problem 1 is affirmative (for both parts) according to the starting remark. Our aim in the course is to analyze (simplify, improve) my proof for Theorem 2, decrease the bound 7 and try to extend it for larger (possibly all) values of $t$.

**Theorem 2.** *Let $\mathcal{H}$ be a 2-intersecting hypergraph with more than two vertices in every edge. Then $\mathcal{H}$ has a 3-strong vertex coloring with at most 7 colors.*

# References

[1] E. Blais. A. Weinstein, Y. Yoshida, Semistrong coloring of intersecting hypergraphs, *arXiv:1203.2868v1 [math.CO] 13 Mar 2012.*