## SZÁMELMÉLET FELADATOK

## KOMPLEX SZÁMOK

- 1. Find the order of the following numbers:  $1, -1, -i, 1+i, (1+i)/\sqrt{2}, \cos(\sqrt{2}\pi)+i\sin(\sqrt{2}\pi), \cos 336^{\circ}+i\sin 33^{\circ}$
- 2. Find the 24th roots of unity and their orders.
- **3.** Adjuk meg az  $x^4+4=0, \quad x^6=1+i, \quad x^n=-1$  egyenletek összes megoldását.
- ${f 4.}$  Find the sum, product and the sum of the squares of the n-th roots of unity.
- **5.** Prove that the product of an n-th and m-th root of unity is a mn-th root of unity. Prove that this product is a primitive nm-th root iff they are both primitive.
- **6.** Multiply the 6-th roots of unity by the 4-th roots of unity. What do we obtain?
- 7. Find  $(\cos 330^{\circ} + i \sin 330^{\circ})^{2001}$  and  $(\cos 30^{\circ} + i \sin 60^{\circ})^{2001}$
- **8.** Find the algebraic form of  $\sqrt{a+bi}$
- **9.** Solve  $x^2 (2+i)x + 7i 1$
- 10. Írjuk föl trigonometrikus alakban:  $1+i, 1-i, -1-i\sqrt{3}, \sqrt{3}+i$
- **11.** Find  $\Phi_n(x)$  for  $n = 12, 72, 144, 100, p^2, p^n, 2p$ .
- 12. Find the sum of the primitive roots of unity (use 5).
- **13.** Find the sum of the primitive roots mod p (use 12).
- 14. Solve  $x^4 + 2 = 8y^2$  among the integers.
- **15.** Solve  $x^4 x^3 + x^2 x + 1 = 3y^2$  among the integers.